

Tree Level Gravity - Scalar Matter Interactions in Analogy with Fermi Theory of Weak Interactions using Only a Massive Vector Field

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Abstract

In this paper we work in perturbative Quantum Gravity coupled to Scalar Matter at tree level and we introduce a new effective model in analogy with the Fermi theory of weak interaction and in relation with a previous work where we have studied only the gravity and its self-interaction. This is an extension of the I.T.B model (Intermediate-Tensor-Boson) for gravity also to gravitational interacting scalar matter. We show that in a particular gauge the infinite series of interactions containing “ n ” gravitons and two scalars could be rewritten in terms of only two Lagrangians containing a massive field, the graviton and, obviously, the scalar field. Using the S -matrix we obtain that the low energy limit of the amplitude reproduce the local Lagrangian for the scalar coupled to gravity.

1 Introduction

In this paper we consider the expansion of the action for a real scalar field coupled to gravity and we study the interaction between scalar particles and gravitons. In the Einstein gauge the interactions contain always two scalars and 1, 2, ... n - gravitons. After the expansion we obtain an infinite number of local Lagrangian terms that contain at n th-order n -gravitons. In analogy with Fermi theory of weak interactions and its extension to the non-local I.V.B model (Intermediate Vector Boson) we introduce only two new Lagrangian terms and we reconstruct all the infinite interactions at tree level in the S -matrix at n -order. This model is an extension of the I.T.B. model [1] to include the scalar matter. In our new model the interaction between n -gravitons and two scalar fields is non-local and it is mediated by a massive spin 1 particle. In the limit in which the mass of the particle goes to infinity we obtain the local interactions of the scalar theory in curved background. We call the model I.T.B (Intermediate Tensor Boson) as in the previous work, also if in this case the interaction is mediated by a vector particle.

2 The I.T.B Model for the Scalar Field

In this section we introduce the I.T.B model for a scalar field coupled to gravity. To do it we introduce two Lagrangian terms, and using only those two terms we reproduce the infinite interaction that contain " n "-gravitons and two scalar fields.

The action for scalar field in curved space-time is

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m^2 \varphi^2] \quad (2.1)$$

The gauge chosen is (see the appendix) :

$$\partial_\mu \sqrt{-g} = 0 \rightarrow \sqrt{-g} = \text{const} = 1 \quad (2.2)$$

Using the developed of the inverse metric $g^{\mu\nu}$ in powers of the graviton $h_{\mu\nu}$ ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$) we have the following form :

$$\begin{aligned} S &= \frac{1}{2} \int d^4x \left[\sum_{n=0}^{\infty} (G_N)^{\frac{n}{2}} (-1)^n (h^n)^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m^2 \varphi^2 \right] = \\ &= \frac{1}{2} \int d^4x \left[\eta^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - (G_N)^{\frac{1}{2}} h^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + (G_N) h^\mu{}_\sigma h^{\sigma\nu} \partial_\mu \varphi \partial_\nu \varphi + \right. \\ &\quad \left. - (G_N)^{\frac{3}{2}} h^\mu{}_\sigma h^\sigma{}_\rho h^{\rho\nu} \partial_\mu \varphi \partial_\nu \varphi + \dots - m^2 \varphi^2 \right] \end{aligned} \quad (2.3)$$

We introduce the "*current*"

$$J^\mu(x) = h^{\mu\sigma} \partial_\sigma \varphi(x) \quad (2.4)$$

This current couples to a spin=1 massive particle as follow

$$L_{(1)} = g_1 J^\mu \Phi_\mu = g_1 h^{\mu\sigma} \partial_\sigma \varphi \Phi_\mu \quad (2.5)$$

This Lagrangian is sufficient to obtain a non local interaction for two gravitons and two scalars. Our purpose is to obtain all interactions between " n "-gravitons and two scalar fields, therefore we introduce a second Lagrangian term that contains a graviton and two spin 1 fields.

$$L_{(2)} = g_2 M_\Phi h_{\mu\nu} \Phi^\mu \Phi^\nu \quad (2.6)$$

Now we calculate the tree level interaction between two gravitons and two scalars in the I.T.B model using the Lagrangian $L_{(1)} + L_{(2)}$, at the second order in the S -matrix

$$\begin{aligned}
S^{(2)} &= \frac{(i)^2}{2} \int d^4x_1 d^4x_2 T[: L_{(1)}(x_1) :: L_{(2)}(x_2) :] = \\
&= \frac{(i)^2}{2} \int d^4x_1 d^4x_2 (g_1)^2 [: (h_{\mu\sigma} \partial^\mu \varphi \overline{\Phi^\sigma})(x_1) :: (h_{\alpha\beta} \partial^\alpha \varphi \overline{\Phi^\beta})(x_2) :] = \\
&= \frac{(i)^2}{2} \int d^4x_1 d^4x_2 (g_1)^2 (h_{\mu\sigma} \partial^\mu \varphi)(x_1) \int \frac{d^4k}{(2\pi)^4} (-i) \frac{\eta^{\sigma\beta} - \frac{k^\sigma k^\beta}{M_\Phi^2}}{k^2 - M_\Phi^2} e^{ik(x_2 - x_1)} \times \\
&\quad \times (h_{\alpha\beta} \partial^\alpha \varphi \overline{\Phi^\beta})(x_2) + \dots \rightarrow \\
&\rightarrow (-i) \frac{(g_1)^2}{2M_\Phi^2} \int d^4x (h_{\mu\sigma} \partial^\mu \varphi h^\sigma{}_\alpha \partial^\alpha \varphi)(x) + \dots
\end{aligned} \tag{2.7}$$

This is exactly the local amplitude that we obtain from the Lagrangian (2.3), provided we make the following identification

$$\frac{(g_1)^2}{2M_\Phi^2} = \frac{G_N}{2} \tag{2.8}$$

The S -matrix to 3^{rd} - order is

$$\begin{aligned}
S^{(3)} &= \frac{(i)^3}{(3!)} \int d^4x_1 d^4x_2 d^4x_3 T[: (L_{(1)} + L_{(2)})(x_1) :: (L_{(1)} + L_{(2)})(x_2) : \times \\
&\quad \times : (L_{(1)} + L_{(2)})(x_3) :] = \\
&= \frac{(i)^3}{(3!)} \int d^4x_1 d^4x_2 d^4x_3 (g_1 h_{\mu\sigma} \partial^\mu \varphi \overline{\Phi^\sigma})_1 (g_2 M_\Phi h_{\alpha\beta} \overline{\Phi^\alpha} \overline{\Phi^\beta})_2 (g_1 h_{\nu\rho} \partial^\nu \varphi \overline{\Phi^\rho})_3 \\
&\rightarrow \frac{(i)}{(3!)} \times 2 \times 3 \frac{g_1^2 g_2}{M_\Phi^4} M_\Phi \int d^4x (h_{\mu\sigma} h_\rho^\sigma h_\nu^\rho \partial^\mu \varphi \partial^\nu \varphi)(x) = \\
&= (i) \frac{g_1^2 g_2}{M_\Phi^3} \int d^4x (h_{\mu\sigma} h_\rho^\sigma h_\nu^\rho \partial^\mu \varphi \partial^\nu \varphi)(x)
\end{aligned} \tag{2.9}$$

In the 3^{rd} line factor 2 comes from all possible contractions of the fields Φ , factor 3 from the fact that the Lagrangian is a sum of two terms and doing the product we obtain a sum of 3 identical factors.

We continue the study of gravity coupled to scalar field going to study the n^{th} -order of the S -matrix. At this order the S -matrix reproduces the interactions between n -gravitons and two scalar fields

$$\begin{aligned}
S^{(n)} &= \frac{(i)^n}{(n!)} \times [(n-2)! 2^{n-2}] \times \frac{n(n-1)}{2} \int d^4x_1 d^4x_2 \dots d^4x_n \times \\
&\quad \times T[: (L_{(1)} + L_{(2)})(x_1) :: (L_{(1)} + L_{(2)})(x_2) : \dots : (L_{(1)} + L_{(2)})(x_n) :] = \\
&= (i)^n 2^{n-3} \int d^4x_1 d^4x_2 \dots d^4x_n \times \\
&\quad \times T[: (L_{(1)} + L_{(2)})(x_1) :: (L_{(1)} + L_{(2)})(x_2) : \dots : (L_{(1)} + L_{(2)})(x_n) :] = \\
&= (i)^n 2^{n-3} \int d^4x_1 d^4x_2 \dots d^4x_n \times \\
&\quad \times (g_1 h_{\mu\sigma} \partial^\mu \varphi \overline{\Phi^\sigma})_1 (g_2 M_\Phi h_{\alpha\beta} \overline{\Phi^\alpha} \overline{\Phi^\beta})_2 \dots (g_2 M_\Phi h_{\gamma\delta} \overline{\Phi^\gamma} \overline{\Phi^\delta})_{n-1} (g_1 h_{\epsilon\tau} \partial^\epsilon \varphi \overline{\Phi^\tau})_n
\end{aligned} \tag{2.10}$$

In the previous equation the multiplying term $[(n-2)!2^{n-2}]$ derives from all possible contractions of the massive fields Φ_μ , while the term $\frac{n(n-1)}{2}$ derives from the fact that the Lagrangian is a sum of two terms and so calculating the product between Lagrangians defined in different points we obtain more copies of the same amplitude.

Now we compare the result obtained from amplitude 2.10 with the local Lagrangian (2.3). We obtain the relation

$$\frac{2^{n-3}}{M_\Phi^n} g_1^2 g_2^{n-2} = \frac{G_N^{\frac{n}{2}}}{2} \quad (2.11)$$

that is valid to all orders in the S -matrix.

If we redefine $g_2 \rightarrow \frac{g_2}{2}$, we obtain

$$\frac{g_1^2 g_2^{n-2}}{M_\Phi^n} = G_N^{\frac{n}{2}} \quad (2.12)$$

For $n = 2$ ed $n = 3$ we have

$$\frac{g_1^2}{M_\Phi^2} = G_N = \frac{1}{M_P^2} \quad (2.13)$$

$$\frac{g_1^2 g_2}{M_\Phi^3} = G_N^{\frac{3}{2}} = \frac{1}{M_P^3} \quad (2.14)$$

and from that :

$$\frac{g_1}{M_\Phi} = \frac{1}{M_P} \quad (2.15)$$

$$g_1 = g_2 \quad (2.16)$$

The question now is if the theory fixes a limit for the mass of the spin 1 field.

The Lagrangian $L_{(1)}$ has adimensional coupling constant that can be very small. The Lagrangian $L_{(2)}$ instead has a coupling constant with mass dimension $g_2 M_\Phi$. Assuming

$$g_1 = g_2 \equiv g \ll 1 \quad (2.17)$$

$$\frac{g_1}{M_\Phi} = \frac{1}{M_P} \quad (2.18)$$

Than

$$M_\Phi \ll M_P \quad (2.19)$$

and if $g_2 \sim 10^{-15}$, we obtain the following mass for Φ :

$$M_\Phi \sim 10^4 GeV \quad (2.20)$$

It is open therefore the possibility to observe quantum gravitational effects at lower energy than Planck scale. In the I.T.B model doesn't exist the Planck scale because the G_N constant has become the mass of a spin 1 particle. It is analog to the I.V.B model of weak interactions where the Fermi constant of weak interactions becomes proportional to the mass of the mediators W^+ , W^- .

3 Outlook and conclusions

In this paper we introduced the I.T.B Model (Intermediate Tensor Boson) [1] for gravity coupled to scalar matter. In this case in the particular suitable gauge $\sqrt{-g} = const$ we expressed all the infinity tree level amplitudes for gravity in the presence of scalar matter using only two Lagrangian

terms. The first Lagrangian term contains a graviton field, a scalar and a vector massive field. The second Lagrangian term contains two spin one massive fields and a graviton. We can write the complete interaction Lagrangian for the I.T.B model as following

$$L_I = g_1 h^{\mu\sigma} \partial_\sigma \varphi \Phi_\mu + g_2 M_\Phi h_{\mu\nu} \Phi^\mu \Phi^\nu - \frac{g_1}{2M_\Phi} h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \quad (3.1)$$

We repeat that only with this interaction Lagrangian we can reproduce all the infinite interaction terms for scalar matter interacting with gravity present in the action (2.3).

In addition we obtain, such in the case of I.T.B model for pure gravity, that the square root of the Newton constant G_N is proportional to the inverse of mediator mass $G_N \sim 1/M_\Phi^2$.

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A The Gauge $\sqrt{(-g)} = \text{const}$

In the I.T.B model for the scalar field we have introduced the gauge $\sqrt{(-g)} = \text{const}$. Now we to study this gauge. This gauge is

$$\Gamma_{\mu\lambda}^\lambda = \frac{1}{2}g^{\lambda\sigma}(\partial_\mu g_{\lambda\sigma}) = \frac{1}{\sqrt{(-g)}}\partial_\mu \sqrt{(-g)} = 0 \quad (\text{A.1})$$

We expand $g_{\mu\nu}$ near the flat metric and we check that it is always locally possible using a diffeomorphism to go from a generic metric to a new metric which satisfies it.

To first order in $h_{\mu\nu}$ the gauge becomes

$$\begin{aligned} \frac{1}{2}\eta^{\lambda\sigma}(\partial_\mu h_{\lambda\sigma}) &= 0 \\ \partial_\mu h^\sigma{}_\sigma &= 0 \end{aligned} \quad (\text{A.2})$$

Under an infinitesimal coordinate transformation the fluctuation transforms as follows

$$\begin{aligned} x'_\mu &= x_\mu + \varepsilon_\mu \\ h'_{\mu\nu} &= h_{\mu\nu} + \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu \end{aligned} \quad (\text{A.3})$$

Using A.3 in A.2, we find

$$\begin{aligned} \frac{1}{2}\eta^{\lambda\sigma}(\partial_\mu h'_{\lambda\sigma}) &= \frac{1}{2}\eta^{\lambda\sigma}(\partial_\mu h_{\lambda\sigma}) + \frac{1}{2}\eta^{\lambda\sigma}\partial_\mu(\partial_\lambda \varepsilon_\sigma + \partial_\sigma \varepsilon_\lambda) \\ 0 &= \partial_\mu h^\sigma{}_\sigma + \partial_\mu \partial^\sigma \varepsilon_\sigma \\ \partial^\sigma \varepsilon_\sigma + h^\sigma{}_\sigma &= \text{cost.} \end{aligned} \quad (\text{A.4})$$

Now we define

$$\varepsilon_\sigma \equiv \partial_\sigma \varphi(x) \quad (\text{A.5})$$

Introducing A.5 in A.4 we get

$$\square \varphi = \text{cost.} - h^\sigma{}_\sigma \quad (\text{A.6})$$

The d'Alambert operator is invertible on L_2 functions, so the solution of A.6 is

$$\varphi(x) = -\frac{1}{\square}(\text{cost.} - h^\sigma{}_\sigma) \quad (\text{A.7})$$

Therefore the condition A.1 is attainable with a diffeomorphism.

To preserve the causality we introduce the Feynman prescription to define the inverse d'Alambertian operator :

$$\square \rightarrow \square + i\epsilon \quad (\text{A.8})$$

Introducing the prescription A.8, the solution of A.6 is

$$\varphi(x) = -\int \frac{d^4 p}{(2\pi)^4} \frac{e^{-ipx}}{p^2 + i\epsilon} [(2\pi)^4 \delta(p) \times \text{cost.} - (\mathcal{F}h)^\sigma{}_\sigma(p)] \quad (\text{A.9})$$

Where $(\mathcal{F}h)^\sigma{}_\sigma$ denote the Furier Transform of $h^\sigma{}_\sigma$.

The Lagrangian for the free graviton in this gauge is

$$L = \partial_\mu h_{\nu\rho} \partial^\mu h^{\nu\rho} - 2\partial_\mu h_{\nu\rho} \partial^\nu h^{\mu\rho} \quad (\text{A.10})$$

While the equation of motion becomes

$$\square h_{\mu\nu} - \partial_\mu \partial^\rho h_{\rho\nu} - \partial_\nu \partial^\rho h_{\rho\mu} = 0 \quad (\text{A.11})$$

The residual gauge invariance (of parameter $\varepsilon(x)$) of A.11 is

$$\square h'_{\mu\nu} - \partial_\mu \partial^\rho h'_{\rho\nu} - \partial_\nu \partial^\rho h'_{\rho\mu} = \square h_{\mu\nu} - \partial_\mu \partial^\rho h_{\rho\nu} - \partial_\nu \partial^\rho h_{\rho\mu} - 2\partial_\mu \partial_\nu (\partial^\rho \varepsilon_\rho) \quad (\text{A.12})$$

To obtain invariance in A.12 we must take the parameter to satisfy

$$\partial_\mu (\partial^\sigma \varepsilon_\sigma) = \text{const.} \quad (\text{A.13})$$

but to preserve the gauge condition A.4 we must take $\text{const} = 0$.

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